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On the interplanetary gas and its magnetic field

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With 3 figures in the text

ABSTRACT

Previously two different models of the interplanetary magnetic field have been proposed by Lüst and Schlüter, and by Alfvén. Lüst and Schlüter have only considered the interaction with the interplanetary gas, whereas Alfvén only considers cosmic ray and magnetic storm data. This paper critically examines both fields and a suggestion for a compromise between the two is made, so that all known physical phenomena of importance are considered. It is also shown that there are three independent arguments why the interplanetary gas density at the earth's orbit cannot be higher than about 0.1 cm^{-3}, viz. (i) cosmic ray data indicate that the gas co-rotates with the sun at the earth's orbit, and no magnetic field is able to maintain this if the density is higher, (ii) beams denser than about 25 cm^{-3} should be seen in the corona according to Alfvén, (iii) beams of a density of about 0.1 cm^{-3} seem to suffice for the production of moderate magnetic storms if the electric field theory is correct.

Introduction

Models of interplanetary magnetic fields have been proposed by Lüst and Schlüter (1954, 1955) and by Alfvén (1956). Lüst and Schlüter's model is based on considerations of the interaction between the magnetic field and the interplanetary gas, which is assumed to co-rotate with the sun in its evironment. Further out from the sun the magnetic field transfers angular momentum to the gas so that the sun's rotation is braked. This may according to Lüst and Schlüter agree with the observed correlation between the rotation and the age (spectral classes) of stars. Alfvén's field, on the other hand, is based on cosmic ray, magnetic storm and auroral data, which Lüst and Schlüter have not considered. Lüst and Schlüter's field is a stationary one, whereas Alfvén's field is "pulsating", expanding due to the action of beams and contracting when the beams cease.

It is clear that when constructing such a magnetic field model one must consider both the interaction with the interplanetary gas and cosmic ray, magnetic and auroral data. Since none of the previous models have done this it appears worth while to make a critical analysis of them and see in what direction they should be modified so as to obtain a compromise which can account for all known physical phenomena in this connection.

According to Ferraro (1937) one should expect that regions which are magnetically connected with the sun, should take part in the sun's rotation. However, near the poles of the sun, there may be magnetic field lines which go to infinity, where they are frozen into matter which is not co-rotating with the sun. Such field lines may be twisted and very complicated phenomena may occur, so we will not consider this. We confine ourselves to regions closer to the equatorial plane where Ferraro's theorem is certainly applicable.

Lüst and Schlüter (loc. cit.) have considered the problem of the size of the co-rotating region. Their result is that the interplanetary gas co-rotates with the sun out to about the orbit of Mercury (0.4 A.U. from the sun). However, they have neglected the centrifugal forces. If these are also accounted for, one finds that the gas cannot co-rotate further out than about 0.2 A.U. from the sun as will be shown below, if the density is as high as Lüst and Schlüter assume. On the other hand, Brunberg and Dattner (1954), Sandström (1956*a*, *b*) and Dattner and Venkatesan (1958) have produced strong arguments from cosmic ray observations that the gas co-rotates even at the earth's orbit at least during periods of high solar activity. If this is correct we are forced to assume a much lower density than has been generally assumed before, as will be shown in this paper. Earlier the estimates of the interplanetary gas density have been based on the theory of the zodiacal light by Behr and Siedentopf (1953). Hence, the results by Brunberg, Dattner, Sandström and Venkatesan seem to be in conflict with the theory of the zodiacal light. However, Behr and Siedentopf's results are based on the assumption that the polarization is only produced by electrons. This is doubtful since polarization may be caused by e.g. elongated grains, lined up by magnetic fields. Also, as pointed out by Van de Hulst (1957), even spherical grains may produce polarization, and he states that more computations are "needed before the astronomical implications can be discussed" (p. 446).

Experiments with whistlers may also give some values of the interplanetary gas density, but at present no certain results seem to have been obtained in this way. Moreover, whistlers can only give densities in the earth's environments (< 5 earth radii from the earth and these may perhaps have little bearings on the densities further out in space.

One of the objects of this paper is to discuss the consequences of Brunberg, Dattner, Sandström and Venkatesan's results and to point out that a reconsideration of the zodiacal light theory is necessary before a more well founded value of the interplanetary gas density can be obtained. The author would like to propose that somebody more familiar with light scattering by small particles should undertake such a reconsideration.

Hydromagnetics of the interplanetary gas

The dynamics of a magnetized gas is determined by

$$\varrho \frac{d\mathbf{v}}{dt} = \mathbf{j} \times \mathbf{B} - \nabla p - \varrho \nabla \phi, \tag{1}$$

where ϕ is the gravitational potential and the other symbols have their usual meaning.

We assume that there is a stationary state with co-rotation within a certain region centered around the sun. It is also assumed that the sun does not eject any matter. Deviations from this will be considered later. Then in a spherical coordinate system in the region of co-rotation equation (1) is reduced to

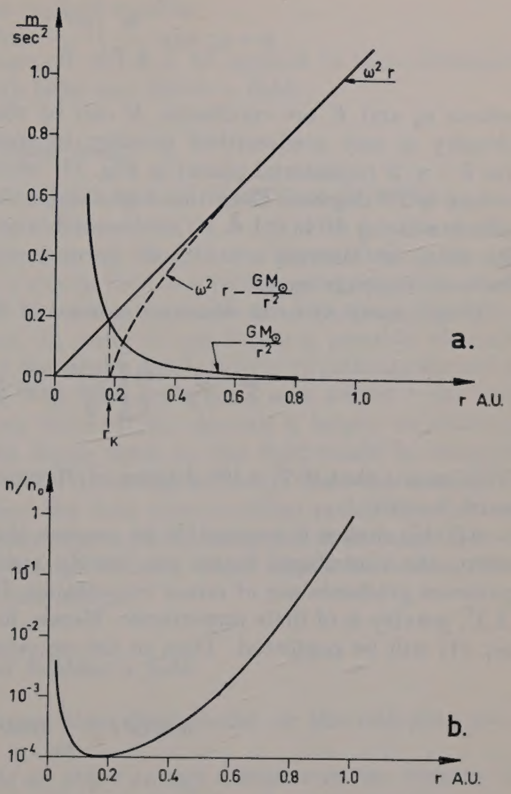

Fig. 1. (a) Centrifugal and solar gravitational forces per unit mass of interplanetary gas as functions of the distance from the sun. The centrifugal force is referred to the equatorial plane only. (b) Relative interplanetary gas density in the equatorial plane as a function of the distance from the sun if there is co-rotation, no magnetic forces are acting and there is thermal equilibrium with temperature of 10^6 °K.

$$-\varrho\,\omega^2 r\sin^2\theta+\frac{\partial p}{\partial r}+\varrho\frac{GM_\odot}{r^2}=(\mathbf{j}\times\mathbf{B})_r, \qquad (1\text{ a})$$

$$-\varrho\,\omega^2 r\sin\theta\cos\theta+\frac{\partial p}{r\,\partial\theta}=(\mathbf{j}\times\mathbf{B})_\theta, \qquad (1\text{ b})$$

$$\frac{1}{r\sin\theta}\,\frac{\partial p}{\partial\varphi}=(\mathbf{j}\times\mathbf{B})_\varphi, \qquad (1\text{ c})$$

where ω = sun's angular velocity, $M_\odot$ = solar mass, G = gravitational constant.

In Fig. 1a $\omega^2 r$ and $GM_\odot r^{-2}$ are shown as functions of r. It is seen that in the equatorial plane for $r < r_k = 0.18$ A.U. (Kepler radius) gravity can keep matter of any density in co-rotation. However, at $r > r_k$ other forces must compensate for the centrifugal forces. Pressure gradients and magnetic forces are then conceivable. Let us first assume that $\mathbf{j}\times\mathbf{B}=0$. Putting $p = nkT$, $\varrho = nm$, where n = proton or electron density (they are approximately equal), $T = T_i + T_e$ is the sum of ion and electron temperatures and m = proton + electron mass, a solution of (1a, b, c) is then easily obtained if T is constant

$$n = n_0 \exp \frac{m}{kT}\left[\frac{\omega^2 r^2}{2} \sin^2 \theta + \frac{G M_\odot}{r} - K\right], \tag{2}$$

where n_0 and K are constants. K can be chosen arbitrarily, so that n_0 can be the density at any predescribed position in space. Equation (2) is shown graphically for $\theta = \pi/2$ (equatorial plane) in Fig. 1b where it has been assumed that the temperature is 10^6 degrees. Even this high temperature requires that the density changes by about a factor 10 in 0.1 A.U. at the earth's orbit, where n has to be about 4000 times its value at Mercury's orbit. At lower temperatures the density gradients would have to be even larger.

If n is assumed to be constant instead of T we obtain

$$T - T_0 = \frac{m}{k}\left[\frac{\omega^2 r^2}{2} \sin^2 \theta + \frac{G M_\odot}{r} - K\right]. \tag{3}$$

This means that if $T = 10^6$ degrees at Mercury's orbit it would be 10^7 degrees at the earth's orbit.

All this makes it reasonable to assume that if there is co-rotation at the earth's orbit, the centrifugal forces are mostly compensated for by magnetic forces, and pressure gradients are of minor importance. It is also seen from Fig. 1 that at $r > 0.2$ A.U. gravity is of little importance. Hence, in the remainder of this paper p and ϕ in eq. (1) will be neglected. Then in the equatorial plane (1a) is reduced to

$$n m \omega^2 r + \frac{1}{\mu_0} (\mathrm{rot}\, \mathbf{B} \times \mathbf{B})_r = 0. \tag{4}$$

In order to obtain easily an approximate expression for the second term in (4) one can often replace it by

$$B^2/2\mu_0 a,$$

where a is the radius of curvature of the field lines. This expression is exact in the case of an infinitely long cylinder with the current parallel to the axis and constant current density (independent of the distance from the axis) but of course it is far too great for any forcefree field where a is finite. However, in any case it is an approximate upper limit, i.e. the true forces are not orders of magnitude larger. Thus (4) can be replaced by the approximate expression

$$\tfrac{1}{2}(n m \omega^2 r^2) \lesssim B^2/2\mu_0 \tag{4a}$$

if it can be assumed that $a \approx r/2$, which is reasonable as an average along substantial parts of the interplanetary field lines. This means that the kinetic energy density due to the rotation cannot be larger than the magnetic energy density. Therefore, if it is known that the matter is taking part in the sun's rotation at a certain distance $r > r_k$ from the sun, and if B is known there, then (4a), or more exactly (4), gives an upper limit of n. Or if n and B are known (4a) gives an upper limit for the radius of the co-rotating volume.

Application to field models

The equations of the preceding paragraph will now be applied to three different fields, a dipole field, Lüst and Schlüter's field and Alfvén's field.

1. *Dipole field*

It may seem unnecessary to apply our equations to a dipole field since this is certainly forcefree so that our equations are reduced to (1a, b, c,) with $\mathbf{j}=0$ and this has already been discussed. However it may be that, if the dipole field is strong enough or the gas density low enough, a dipole field is a rather good approximation to a field which is able to keep the gas in co-rotation with the sun even if gravity and pressure gradients can be neglected. In order to see if this is possible we may apply (4a) to a dipole field. We then find that if the dipole field is 12 gauss at the sun's poles the density must be $n \ll 10^{-2}$ cm^{-3} at Mercury's orbit and $n \ll 10^{-5}$ cm^{-3} at the earth's orbit if the gas is co-rotating there. If the density is larger, co-rotation would cause a strong disturbance of the dipole field, i.e. the field would be dragged out considerably by the centrifugal forces. It seems unlikely that the density is so low, and hence, we may conclude that the field always differs rather much from a dipole field if co-rotation is the normal state out to some distance from the sun larger than, say, 0.4 A.U. (Mercury's orbital radius), which seems to be the case according to Dattner and Venkatesan (loc. cit.).

2. *Lüst and Schlüter's field*

Lüst and Schlüter (loc. cit.) have based their field model on the following considerations.

The magnetic energy density exceeds all other energy densities in the vicinity of the stars, including the sun. This means that the theorem of isorotation (Ferraro, 1937) is applicable here, so that in a steady state when matter is co-rotating with the sun, the moment of momentum of the magnetic field must vanish in this region, i.e. $\mathbf{B} \times \operatorname{rot} \mathbf{B}$ must vanish in the φ-direction. Since all other energy densities are assumed to be very small in comparison to the magnetic energy density, the other components of $\mathbf{B} \times \operatorname{rot} \mathbf{B}$ should be regarded as small, and therefore a forcefree field should be a good approximation to the real field sufficiently close to the sun. On the other hand, it is very probable that the sun's rotation has slowed down during its lifetime, as pointed out in the beginning of this paper. According to Lüst and Schlüter the rotation has probably been braked by the magnetic field, which therefore must be able to transport angular momentum. It is assumed that this process is still going on. However, they were not able to construct an entirely forcefree field, which could do this, but a suitable moment of momentumfree field was found. This field, which we will call B_l-field, is given by

$$B_{rl} = 2(P_0/r^2)\cos\theta, \tag{5}$$

$$B_{\theta l} = 0, \tag{6}$$

$$B_{\varphi l} = -(1/2r)\,T_0 \sin\theta\cos\theta. \tag{7}$$

$2\pi P_0$ is equal to the total flux of the field and T_0 is such that $B_{rl} \approx B_{\varphi l}$ at $r = r_m$, where r_m will be defined after eq. (11).

This field is forcefree not only in the φ-direction but also in the r-direction, so the force has only a θ-component.

$$F_{\theta i}=\frac{\sin\theta\cos\theta}{\mu_0 r^3}\left\{\frac{4P_0^2}{r^2}-\frac{T_0^2}{4}(3\cos^2\theta-1)\right\}. \tag{8}$$

It is transporting a moment of momentum

$$\tau=(8\pi/15\mu_0)P_0T_0 \tag{8a}$$

from the origin to infinity.

This field must be matched to a field in the interior of the sun which is not moment of momentum free but can take up angular momentum from the sun. Outside a region approximately given by (4) or (4a) it must be matched to another field which can give off angular momentum to the interplanetary gas. This gas is certainly not co-rotating. The external field (B_e-field) is given by

$$B_{re}=2P_0\cos\theta\frac{r_m}{r^3}\left(2-\frac{r_m}{r}\right), \tag{9}$$

$$B_{\theta e}=2P_0\sin\theta\frac{r_m}{r^3}\left(1-\frac{r_m}{r}\right), \tag{10}$$

$$B_{\varphi e}=-\frac{T_0}{2}\frac{r_m^2}{r^3}\sin\theta\cos\theta. \tag{11}$$

Here r_m is the radius of a sphere, on the surface of which the two fields are matched together. Thus, at $r=r_m$ the following conditions are fulfilled.

$$(\mathbf{B}_i)_{r=r_m}=(\mathbf{B}_e)_{r=r_m}, \tag{12}$$

$$(\mathbf{r}\cdot\operatorname{rot}\mathbf{B}_i)_{r=r_m}=(\mathbf{r}\cdot\operatorname{rot}\mathbf{B}_e)_{r=r_m}. \tag{13}$$

Hence, even the external field lines derive from the sun. To this field still another (galactic) field, $\mathbf{B}_g$, may be matched, which is not of solar origin, e.g. a field which is homogeneous at infinity. This matching can be done on a sphere with radius $r_g > r_m$. Outside $r\approx r_g$ the matter should be at rest. The angular momentum transported by $\mathbf{B}_i$ is consumed by $\mathbf{B}_e$ outside $r=r_m$ and transported from there by turbulence towards infinity, where the B_g-field prevails.

It is now our object to discuss the value for r_m and the gas density. According to Lüst and Schlüter this is determined by

$$(B_i^2/2\mu_0)_{r=r_m}\approx\alpha^2\cdot\varrho\,\omega^2 r_m^2, \tag{14}$$

where α is a proportionality factor, such that the right hand side of (14) gives the kinetic energy density due to the turbulence in the gas. A suitable value for α^2 would be $\frac{1}{50}$ according to Lüst and Schlüter. It is difficult to understand this, since (14) gives then a value for r_m, which is about twice as large as that obtained from (4a) if ϱ does not change with r, and if the field according to (5)–(7) is used. Then (14) gives

$$r_m\approx(P_0/\alpha\,\omega)^{1/3}\cdot(\varrho\,\mu_0)^{-1/6}) \tag{15}$$

and (4a) corresponds to $\alpha = 1$. Thus it seems to be more correct to compare the magnetic energy density with the kinetic energy density due to the co-rotation rather than that due to turbulence.

Using (15) with $P_0 = 10^{14}$ Wb, corresponding to a polar field strength of about 4 gauss and $\varrho = 10^{-17.5}$ kg/m³, corresponding to 2×10^3 protons/cm³, Lüst and Schlüter find $r_m \approx 10^{10.8}$ m, but if we use (4a) instead, we get $r_m \approx 10^{10.5}$ m, i.e. almost exactly equal to r_k.

The external field, given by (9)–(11), gives rise to forces $\mathbf{F}_e$ on the gas.

$$F_{re} = \frac{1}{\mu_0} \sin^2\theta \, \frac{r_m^3}{r^7} \left\{ \frac{T_0^2}{2} r_m \cos^2\theta - \frac{8 P_0^2}{r} \left(1 - \frac{r_m}{r}\right) \right\}, \tag{16}$$

$$F_{\theta e} = \frac{1}{\mu_0} \sin\theta \cos\theta \, \frac{r_m^3}{r^7} \left\{ \frac{T_0^2}{4} r_m (1 - 3\cos^2\theta) + \frac{8 P_0^2}{r} \left(2 - \frac{r_m}{r}\right) \right\}, \tag{17}$$

$$F_{\varphi e} = \frac{1}{\mu_0} \sin\theta \, \frac{r_m^3}{r^7} P_0 T_0 \left\{ 1 + \cos^2\theta - \frac{r_m}{r} \sin^2\theta \right\}. \tag{18}$$

The r-component of this force in the equatorial plane is reduced to

$$F_{re} = -\frac{1}{\mu_0} \frac{8 P_0^2}{r_m^5} x^8 (1 - x) \tag{16 a}$$

with $x = r_m/r$.

The maximum of $|F_{re}|$ in the equatorial plane occurs at $x = \frac{8}{9}$.

$$|F_{re}|_{\max} = \left(\frac{8}{9}\right)^9 \frac{P_0^2}{\mu_0 r_m^5}. \tag{16 b}$$

Introducing this in (4) we find $n \approx 2$ protons/cm³ if we take Lüst and Schlüter's value for r_m ($10^{10.8}$ m) and $n \approx 100$ protons/cm³ if $r_m = 10^{10.5}$ m, as given by (4a). Thus, the magnetic field cannot compensate for the centrifugal forces outside $r = r_k$ if the density is as high as Lüst and Schlüter assume.

A solution of (1a, b, c) if $\mathbf{j} \times \mathbf{B}$ is replaced by $\mathbf{F}_i$ according to (8) at $r < r_m$ and by $\mathbf{F}_e$ according to (16)–(18) at $r > r_m$ is not possible since neither $\mathbf{F}_i$ nor $\mathbf{F}_e$ are conservative forces. Hence (1) can be satisfied only if a suitable velocity field is introduced which can cansel out the non-conservative part of $\mathbf{j} \times \mathbf{B}$. Such a solution may be found, which includes a continuous stream of matter outwards in all directions from the sun. This seems to be the only possibility to maintain the assumption of a density as high as Lüst and Schlüter's. It is known that the interstellar gas density between clouds as an average is not more than 0.1 particles/cm³ (Allen, 1955, p. 225). One should expect that the density in a whirl, like the co-rotating region around the sun, should be lower or at least not higher than in the surroundings, if there is no source of matter inside the whirl. Now, the sun may be such a source so the density can be higher, but Lüst and Schlüter's density requires an ejection of matter from the sun with a density of the order of 10^7 cm^{-3} in the corona. Such a dense stream should be seen optically, and there is so far no indication of this as pointed out by Alfvén (1957). It should also cause strong continuous aurorae on the earth, because it is probable that beams with a density of more than 1 particle/cm^3 would cause very strong aurorae (Block, 1956).

Because of the co-rotation it is not unreasonable to assume that the density at the earth's orbit is lower than in interstellar space. As will be seen later there are arguments for the assumption that the density at the earth's orbit is not higher than 0.1 cm^{-3}.

Alfvén's field

First we want to evaluate the maximum gas density compatible with the assumption of co-rotation in Alfvén's field as a function of the distance from the sun. For this we have to use eq. (4).

The equations of the field are given in a paper by Block (1958). The field is divided into five different regions, only two of which are of interest here. Region II is the beam region, and region III is the recovery region, where the field is recovering to its initial state after the beam has passed by (Fig. 2). We give the equations of the field in these two regions.

Region II:

$$B_r = -B_0\left(\frac{r_\odot}{r}\right)^2 \cos\theta + K_1 \frac{r - r_\odot}{r^2} \cot\theta, \tag{19}$$

$$B_\theta = -K_1 r^{-1}, \tag{20}$$

$$B_\varphi = 0. \tag{21}$$

Region III:

$$B_r = -B_0\left(\frac{r_\odot}{r}\right)^2 \cos\theta + K_2 r^{-2} \cot\theta \log\frac{r}{r_\odot}, \tag{22}$$

$$B_\theta = -K_2 r^{-2}, \tag{23}$$

$$B_\varphi = 0. \tag{24}$$

These equations are valid only in the latitudes between 20° N and 20° S. ($70° < \theta < 110°$.) Further

$$B_0 = 1.2 \times 10^{-3}\ \mathrm{Wb/m^2},$$

$$K_1 = 0.65 \times 10^{-6}\, r_\odot\ \mathrm{Wb/m},$$

$$K_2 = 0.88 \times 10^{-4}\, r_\odot^2\ \mathrm{Wb},$$

and $r_\odot$ is the sun's radius measured in meters.

The calculation will be performed only in the equatorial plane where $\sin\theta = 1$. Calculations of the forces and corresponding densities at other latitudes according to (4) show that they are of the same order as those in the equatorial plane. There are also surface currents at the boundaries between the different regions, but the forces due to these surface currents are also of the same order (or usually smaller by a factor 2 or so) as the integral of the volume forces over the beam latitudes (20° N–20° S).

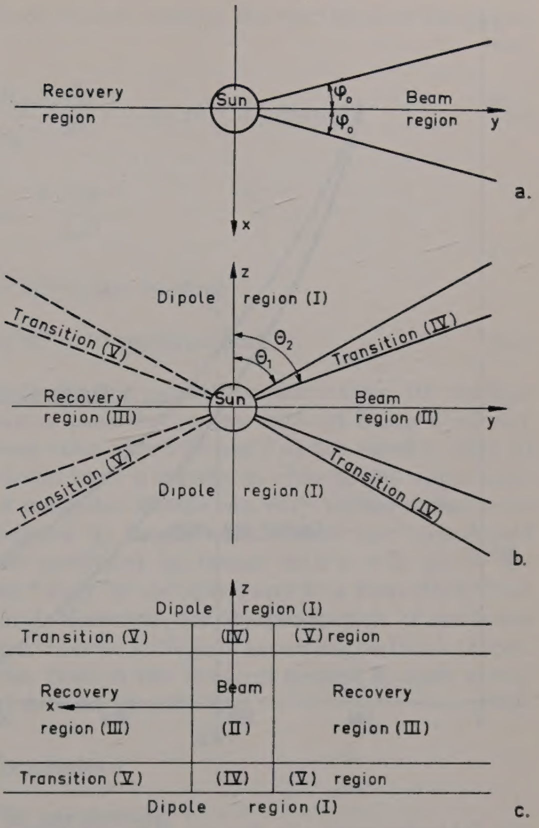

Fig. 2. The different regions of Alfvén's interplanetary magnetic field. (*a*) View from above the northern hemisphere of the sun. (*b*) A meridional plane. (*c*) View inside the beam towards the sun.

In this case we have
$$F_r = -\frac{1}{\mu_0}\,(\text{rot}\,\mathbf{B})_\varphi \cdot B_\theta \tag{25}$$

since $B_\varphi = 0$ in Alfvén's field.

In region II (beam region) we obtain from (19)–(21)

$$F_r^{\text{II}} = \frac{K_1}{\mu_0 r^4}\left\{K_1 \frac{r - r_\odot}{\sin^2\theta} - B_0 r_\odot^2 \sin\theta\right\}, \tag{26 a}$$

and in region III (recovery region) equations (22)–(24) give

$$F_r^{\text{III}} = \frac{K_2}{\mu_0 r^5}\left\{K_2\left(1 + \operatorname{cosec}^2\theta \log\frac{r}{r_\odot}\right) - B_0 r_\odot^2 \sin\theta\right\}. \tag{26 b}$$

In these two equations $B_0 r_\odot^2 \sin\theta$ dominates over the other terms out to several A.U.'s from the sun.

Introducing (26a, b) in (4) and putting $\sin\theta = 1$ we can solve for n. The results are shown in Fig. 3. The values for n so obtained are upper limits in region III for

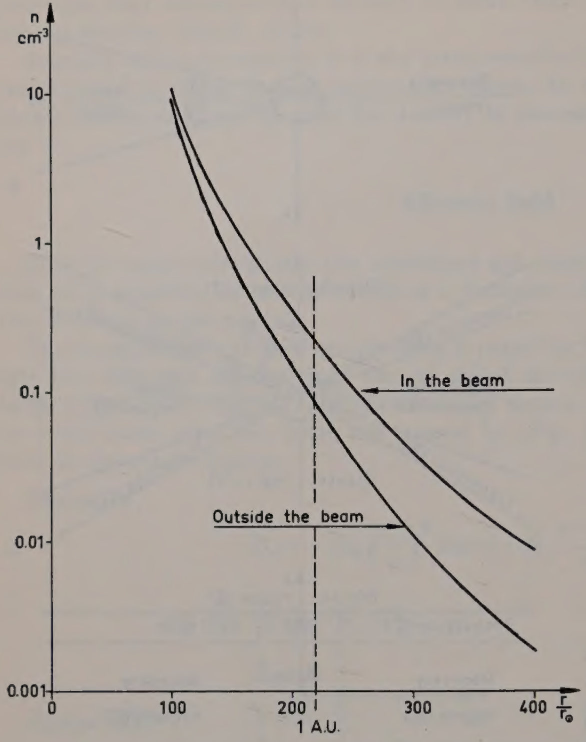

Fig. 3. The maximum proton density in space which can be kept in co-rotation in the equatorial plane by Alfvén's field as a function of the distance from the sun.

the interplanetary gas density if co-rotation really takes place. In region II on the other hand, the density may be larger since matter is moving outwards in the beam and the magnetic field is not able to contract there.

From Fig. 3 it can be seen that a maximum density of about 0.1 particles/cm^3 can be allowed at the earth's orbit if co-rotation takes place.

The magnetic forces in the beam region may act as a brake on the beam if the beam density is too low. We may therefore calculate a minimum density in the beam which is compatible with the present magnetic field model. It seems resonable to assume that the integral of the magnetic force from the sun to the earth should not very much exceed, say 50 % of the initial kinetic energy of the beam at the sun. In computing this one must account for the spread of the beam on its way from the sun. Considering one m^3 of gas at the earth's orbit, this gas occupies a volume of only (r^2/R^2)m^3 at the distance r from the sun, R being the sun–earth distance. Since all particles move with the same velocity (they are all in the same crossed electric and magnetic fields) there is no longitudinal spread so the force F_r in the integrand should be multiplied by r^2/R^2. Thus we obtain

$$\int_{r_\odot}^{R} -F_r^{\mathrm{II}} \frac{r^2}{R^2}\, d r \leqslant 0.5 \frac{n\, m\, v^2}{2}. \tag{27}$$

The force F_r is taken from (26a) where we may neglect the first term in the parenthesis.

$$\int_{r_\odot}^{R} \frac{K_1 B_0 r_\odot^2}{\mu_0 r^4} \cdot \frac{r^2}{R^2}\, dr \approx \frac{K_1 B_0 r_\odot^2}{\mu_0 r_\odot R^2} = 1.5\times 10^{-8} \text{ Joules/m}^3. \tag{28}$$

Thus

$$n \geqslant \frac{6\times 10^{-8}}{m v^2}.$$

With $m = 1.67 \times 10^{-27}$ kg and $v = 2 \times 10^6$ m/sec we find

$$n \geqslant 10^7 \text{ particles/m}^3 = 10 \text{ particles/cm}^3. \tag{29}$$

According to Alfvén (1957) the beam density cannot be more than 10^6 cm^{-3} in the corona, because denser beams cannot pass the corona without being observed. This means that beams cannot be denser than about 25 cm^{-3} at the earth's orbit. It is not likely that the weakest beams should have a density so close to the upper limit as (29) indicates, since the intensity of magnetic storms can vary within rather wide limits. If the weakest storms are produced by beams with $n \approx 10$ cm^{-3} one should certainly expect that heavy storms are produced by beams with $n > 25$ cm^{-3}. According to Alfvén (loc. cit.) even 25 cm^{-3} may be too much and it is more likely that the upper limit is of the order of 2 cm^{-3}. Moreover, for the production of moderate magnetic storms a density of only 0.1 cm^{-3} seems sufficient according to Block (1956). Hence, it is probable that the magnetic field in the beam is shaped in such a way that it is more forcefree than the field defined by (19)–(21).

Conclusions

1. *The gas density*

Table 1 gives a summary of some of the results obtained in the preceding paragraphs. The densities are obtained by eq. (4) or (4a).

According to these results it seems very difficult, if not impossible, to construct a field which is able to keep an interplanetary gas in co-rotation with the sun at the earth's orbit if the density is substantially higher than 0.1 cm^{-3}, e.g. 100–1000 cm^{-3} as required by Behr and Siedentopf's zodiacal light theory. In order to obtain such a field we would have to assume a solar field with several thousand gauss at the poles.

Table 1.

Distance from the sun	Dipole field (12 gauss at sun's pole)		Lüst and Schlüter (4 gauss at sun's pole)		Alfvén (12 gauss at sun's pole)	
	B gauss	n cm^{-3}	B gauss	n cm^{-3}	B gauss	n cm^{-3}
0.4 A.U. (Mercury)	10^{-5}	10^{-2}	10^{-4}	2	10^{-4}	20
1.0 A.U. (Earth)	6×10^{-7}	10^{-5}	—	—	$2\text{–}3\times10^{-5}$	0.1

Hence, if we accept Sandström's and Dattner and Venkatesan's result about the co-rotation, we have to assume an interplanetary gas density at the earth's orbit not higher than 0.1 cm^{-3}. Further arguments for this assumption are mentioned before, viz. that beams cannot be denser than at most 25 cm^{-3} as pointed out by Alfvén (1957), and that beam densities of 0.1 cm^{-3} seem to suffice for the production of moderate storms according to Block (1956). Although it is difficult to make a definite statement it seems likely that beams should be denser at the earth's orbit than the surrounding interplanetary gas.

2. *The geometry of the field lines*

Alfvén (1956) has based his field on the following assumptions.

(i) If there were no beams the field would be a dipole field in a stationary state.

(ii) Beams have a tendency to drag out the field lines from the sun. Considering frozen-in field lines in the beams, one arrives at the conclusion that the field in the beams should be inversely proportional to the distance from the sun.

(iii) When a beam has passed over a certain region of space, the field recovers towards a dipole field so in these regions the field may e.g. be inversely proportional to the square of the distance from the sun, or something like that.

There are three objections against this field.

(*a*) It is seen from the figures for the dipole field in table 1, that such a field is not compatible with Dattner and Venkatesan's result about co-rotation, because there are no reasons to assume that the gas density should be as low as 10^{-5} cm^{-3} the earth's orbit. Hence, assumption (i) is not valid.

(*b*) As described in a previous section the magnetic forces of the field act as a brake on the beams so that a minimum density corresponding to 10 cm^{-3} at the earth's orbit is required in the beams if they should be able to reach the earth. This appears to be too high so it is likely that the field is more forcefree.

(*c*) In Alfvén's field it has been assumed that the beam is radial out from the sun, but in nature it is curved because of the sun's rotation. This makes it plausible, that also the magnetic field lines are curved instead of confined in the meridional planes. In fact, one should expect that where ever matter is moving outwards from the sun, the magnetic field lines are curved in the same direction as the beam, and where ever matter is moving inwards towards the sun, the field lines are curved in the opposite direction. In other words, we expect that where ever there is a radial motion, there should also be toroidal field components. An illustration of this is the following.

According to Eckhartt (1958) it was possible to determine the direction of incidence of the cosmic rays causing the intensity increase on Feb. 23, 1956. This direction was not from the sun but from a beam which caused a magnetic storm on Feb. 25. As pointed out by Block (1958) this may be explained most easily if the field lines were curved, so that there was a toroidal component of the field (cf. Block, 1958, Fig. 8).

It will be investigated if these objections may be met with by making some small changes in Alfvén's field so that it is more like Lüst and Schlüter's field.

Lüst and Schlüter's field is not subject to objection (*a*). The B_i-field (see eq. (5)–(7)) is forcefree in the r-direction but the B_e-field can compensate for centrifugal forces if the density is not too high. Hence, it is reasonable to replace the dipole field according to assumption (i) in Alfvén's field by a field which is more like Lüst and Schlüter's field. The latter is not decreasing with increasing r as much as a dipole field. It has also the advantage of being more forcefree than Alfvén's field in the equatorial regions, especially the B_i-field (eq. (5)–(7)) close to the sun where the beams are ejected. Thus, neither is it subject to objection (*b*). Also, the field has toroidal components as required by objection (*c*). However, the sign of the toroidal component (eq. (7)) is opposite to what it should be to explain Eckhartt's cosmic ray impact direction in Feb. 1956. On the other hand, the B_i-field is very weak in the equatorial plane so if this field was extended out to the earth's orbit on Feb. 23, 1956, it does not matter if the field lines are curved in the opposite direction. Then Eckhartt's explanation of the cosmic ray impact direction may be valid (cf. Eckhartt, loc. cit., and Block, 1958, p. 174). However, there is also another possible explanation which will be described now.

If we for a moment accept Lüst and Schlüter's field during the absence of beams, we must consider how this field is changed when beams are switched on, and also the consequences for cosmic rays and magnetic storms.

The field may be described in a stationary state by the equations (5)–(7) (B_i-field) at $r < r_m$ and by (9)–(11) (B_e-field) at $r_m < r < r_g$. In the B_i-field there is complete co-rotation, and in the B_e-field there is incomplete co-rotation and perhaps turbulence. At $r > r_g$ there is no co-rotation at all.

If a beam is ejected from the sun the radial components of the field should be affected very little, but the θ- and φ-components should be pushed out. As a consequence one may expect that when the solar activity has been high for some time r_m has increased somewhat. Hence, the region of co-rotation has increased in volume. During this process there has been a transport of angular momentum outwards from the vicinity of the sun. When the beams are switched off because of decreasing solar activity the field resets and r_m decreases again. Then in the B_e-region there is an inward transport of angular momentum because also the co-rotating gas moves inward together with the magnetic field. However, an inward transport of angular momentum corresponds to an opposite sign of the toroidal component, which means that the curvature of the field lines agrees with Eckhartt's impact direction of cosmic rays. If this interpretation is correct the toroidal field had already changed its direction after the preceding beam passed the earth before Feb. 23, 1956. The preceding storm occurred on Feb. 11–12, so 11 days are then sufficient for the field to change direction.

Dattner and Venkatesan (loc. cit.) claim that there is co-rotation at the earth's orbit during high solar activity because then there is an 18-hour component in the anisotropy of the cosmic radiation. This decreases or even vanishes during low solar activity. An interpretation of this in terms of Lüst and Schlüter's field would be that $r_m \gtrsim 1$ A.U. at high solar activity and $r_g < 1$ A.U. at low solar activity. Such a change in volume of the co-rotating region must necessarily be accompanied by a radial transport of an angular momentum, at least in the vicinity of $r = r_m$.

Toroidal field components will change the direction of the electric fields in the beams which cause magnetic storms and aurorae. During high solar activity when there may be a B_i-type field at the earth, the θ-component of the magnetic field should be very small (eq. (6)) so the electric field should then more often be parallel to the earth's axis than at other times. It would therefore be of great interest to analyse magnetic storms to see whether any storms compatible with an electric field parallel to the earth's axis ever occurs. Such storms would be characterized by currents in the ionosphere along meridians from one pole to another. The currents would be stronger on the dayside than on the nightside of the earth because of the higher conductivity in the ionosphere on the dayside. The direction of the currents (the electric field) would depend both on the direction of angular momentum transport and on the position of the earth in relation to the equatorial plane of the interplanetary field, because $B_{\varphi i}$ changes sign at $\theta = 90°$ according to (7). However, it should be noticed that the direction of curvature of the field lines only depends on the direction of angular momentum transport.

It has also been noticed by Venkatesan (1957) that the electric field causing magnetic storms may be completely reversed in some cases, beacuse then high K_p-index is associated with a rise in cosmic ray intensity instead of with a fall. This may be due to either a reversed magnetic field or a reversed direction of the cosmic wind such that matter is falling back towards the sun. Then the magnetic field should also contract. Magnetic storm data should be investigated with the aim of discriminating between these two possibilities. If the latter is true one should in particular expect storms with reversed electric field during falling solar activity. According to Venkatesan (personal communication) he has found such storms in 1951 (shortly after a peak in solar activity) on May 26, July 29, July 31–Aug. 2, Sept. 16 and Sept. 19–22. The two first of these were preceded 2 or 3 days earlier by normal storms.

In view of all this it seems probable that mostly $r_m < 1$ A.U. and $r_g > 1$ A.U. so that there is some but not complete co-rotation at the earth's orbit. On a few occasions with extremely low solar activity as in 1954 also $r_g < 1$ A.U. so that there is no co-rotation here. On the other hand at or just after a period of extremely high solar activity $r_m > 1$ A.U. so that there is complete co-rotation. A closer study of cosmic ray, magnetic storm and auroral data may reveal whether this is true, and if so at what approximate times r_m and r_g pass the earth's orbit.

It must be pointed out that the difference between the field proposed here and Alfvén's field is not very great. One could characterize the present field by saying that it is, either an Alfvén field where a toroidal component has been added in order to account for the transport of angular momentum as described by Lüst and Schlüter, or a Lüst-and-Schlüter field where the influence of beams has been added so that the field pulsates in and out as described by Alfvén.

The question about the slowing down of the sun's rotation was the initial reason why Lüst and Schlüter proposed their field. Whether this is still going on or not is an open question. Lüst and Schlüter seem inclined to assume so, although they say that the rate of slowing down is extremely critically dependent on the initial gas density around the sun. A change of this density by a factor 10 would change the slowing down time by several powers of ten. Alfvén (1954, p. 187) on the other hand says that the initial angular momentum of the sun may have been transferred to the planetary system when this was formed. Then swiftly rotating stars would be so hot that they cannot form planetary systems, whereas slowly rotating stars are cold enough to be able to do this (cf. Alfvén, 1954). The slowing down time would be short in comparison to the age of the planetary system.

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